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POWER SPECTRUM ANALYSIS OF CLIMATOLOGICAL DATA FOR WOODSTOCK COLLEGE, MARYLAND

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ABSTRACT

Power spectrum techniques are applied to series of daily, weekly, and monthly average temperature and precipitation values, recorded since 1870 at the Woodstock Climatological Benchmark Station in Maryland, in order to gain a reasonable interpretation of the extent and frequency distribution of periodic variations in these data. Analysis procedures are outlined, and the results presented, interpreted, and collated with the results of earlier literature in some detail.

Apparent short-period variations are found whose periods lie near 3 days, between 5 and 7 days, and between about 15 and 25 days; various of them however, are absent from some portions of the data series and also differ somewhat in character with the season of the year.

Significant long-period variations are more prevalent in the temperature series than in the precipitation series. Spectral peaks in temperature, of periods near 2 years and greater than 50 years, both achieve high levels of statistical significance. The 11-year sunspot cycle, and to some extent its second harmonic as well, is suggested in the temperature data. The double (22-year) sunspot cycle and the longer Brückner cycle, however, are almost totally absent. The basis of Abbot's statistical long-range prediction scheme, which utilizes numerous higher harmonics of the double sunspot cycle, is tested against the Woodstock data, and is found in this case to lack measurable skill above chance.

1. INTRODUCTION TO PROBLEMS OF WEATHER PERIODICITIES

The notion of periodic phenomena in weather conditions is probably as old as mankind. The scientific search for such cyclic elements in weather and climate dates back to the days of the first instrumental observations of atmospheric events. The underlying idea was borrowed from astronomy. In celestial mechanics it had been eminently successful to resolve the complicated motions of the sun, the moon, and the planets into *ephemerides* which predicted with high accuracy their positions, movements,

and occultations. The early scientists tacitly concluded that the weather must be arranged in some lawful order also: why not analyze it the same way as planetary observations? Thus the almanac was born with its juxtaposition of celestial and weather predictions.

But weather did not quite fit the pattern. Thus ever since the middle of the 18th century studies on cycles in the atmosphere have accumulated. Some exhibit wishful thinking, many others are serious. One of the most recent extensive studies (Berlage [7]), dealing with cycles over one year in length, lists 55 periods ranging from 1.03 to 36

years. These have been proclaimed by various investigators as present in meteorological data series. Berlage lists over 350 references, nearly all of them in the literature of the current century. Numbers of the same order of magnitude could be given for cycles in the shorter time interval of less than one year. It is interesting to note that most modern writers in the field refer to *hidden* periodicities. This implicitly admits that the regular variations are masked by meteorological "noise", or apparently random fluctuations. But authors of pertinent papers have made more or less modest claims for the forecasting value of alleged periodicities (Wing [38]). In some recent writings this has led to attempts at extrapolation into the future of long rainfall series in the Middle West and Southwest of the United States (Abbot [1]).

What reasoning maintains the belief in weather cycles? Most important is, of course, the actual presence of a few atmospheric periodicities. The diurnal and annual variations produced by axial rotation of the earth and the planet's revolution around the sun are so pronounced in their effects that it seems almost superfluous to mention them. In these cases an overwhelming "forcing function" is operative and results can be readily expressed as harmonic elements in the diurnal and annual march of most meteorological variables. Also present, without doubt but with infinitesimally smaller amplitude, are periodic manifestations of lunar tidal forces in the atmosphere. In fact, it took considerable scientific detective work to discover the true lunar gravitational tides. Their effect on pressure is generally only a fraction of a millibar (Chapman [10]). As in any mechanical system one should also expect in the atmosphere free oscillations. These are primarily governed by the vertical temperature distribution and stratification. Their order of magnitude is from fractions of an hour, in a local setting, to fractions of a day for the whole atmospheric layer.

Interestingly enough, the kinematics of the atmospheric circulation produces other quasi-periodic elements in the field of atmospheric motions. For shorter time intervals of a few days these have been made theoretically plausible (Rossby [26], Haurwitz [16]). Somewhat longer cyclical elements also become apparent in the calculations involving simplified theoretical models of the general circulation (Smagorinsky [31]).

Somewhat overlooked in recent years have been the suggestions that the arrangement of the subtropical anticyclones indicates fairly stable vorticity concentrations, which in turn could produce periodic oscillations (Stewart [32]). Rough estimates of the normal modes of such oscillations give the wide brackets of 2,000 to 5,000 days and 70,000 to 250,000 days. The arrangement of the fixed geographical features of the earth also suggests that there would be an induction of periodic elements into atmospheric motions, both on a small and a large scale.

Probably the widest discussion has ranged over the possibility of forced fluctuations in atmospheric param-

eters because of variable solar radiation. These have been primarily associated with the solar period of rotation of 27 days and the irregular sunspot rhythm of 11.3 years, its fractions and its multiples. The tantalizing circumstance here is that there are close correlations between the solar conditions and the ionospheric responses. But a strong link to the lower atmosphere has still to be forged. The best one can say is that some evidence of solar influences, other than the diurnal and annual, exists. But these influences wax and wane quite irregularly. Typical of the results obtained is a recent analysis of the 27-day cycle in terrestrial temperature data for limited intervals by Visser [36].

Berlage [6, 7] in his comprehensive treatises tries to make a case for a combination of solar and terrestrial factors leading to the multiplicity of observed periodicities. A primary element in his system is the late Sir Gilbert Walker's *Southern Oscillation*. This is the fluctuation of the pressure difference between the Malayan Archipelago and Easter Island. It is argued that this is a primary terrestrial period of $2\frac{1}{3}$ (2 to 3) years, caused by mutual interactions of air and sea temperatures, the latter influenced by oceanic currents. Lower harmonics of this "cycle", ≈ 5 and ≈ 7 years, then show interference with solar cycles (say perhaps, $5.6 = \frac{1}{2}$, $11.3 = 1$, $23 = 2$ solar periods). This leads to beat frequencies and the confusing welter of meteorological time series. Other basic cycles might, of course, participate in the merry rhythm dance.

It is, therefore, understandable that a great deal of cycle research has been undertaken essentially in quite an empirical fashion, with the hope of first obtaining statistically significant results and then, if possible, of tying these to some plausible physical cause. Most of these studies have used some form of harmonic or periodogram analysis. A review of the findings which are based on reasonably adequate statistical procedures reveals as the most universal rhythms the following: $2\frac{1}{3}$, $3\frac{1}{3}$, 5-6, 11-12, 19-24, and 30-35 years in length (Landsberg [18]).

The longer the span the more irregular is the interval between extremes. These rhythms were, however, noted in such diverse elements as pressure, temperature, precipitation, lake levels, temperature range, and extreme weather conditions in series of observations originating at such distant points as North America, Central and Eastern Europe, and Indonesia.

Panofsky and McCormick [24] have stated, "Direct harmonic analysis yields a number of harmonics equal to half the number of observations. The amplitude of these harmonics oscillates wildly from one harmonic to the next. These oscillations are not reproducible from one time series to another which has basically the same statistical properties. We, therefore, need to compute a smooth spectrum. The autocorrelation method with the number of lags small compared to the number of observations yields a smooth spectrum directly with a great deal less

numerical work than would be required for computing and smoothing the spectral estimate obtained by direct Fourier analysis."

It is always good scientific practice to use new procedures and additional data to probe into unresolved puzzles. For time series of rapidly fluctuating elements power spectrum analysis has become a widely accepted technique to separate "signals" from "noise." In meteorological series this tool has been primarily used for short-period phenomena, such as turbulent wind fluctuations. It has gradually found its entry into analysis of wave motions on a hemispheric-synoptic scale. We will attempt to extend its use here to climatic series. As object of this analysis we have chosen the climatic reference station at Woodstock, Md. For this station daily, weekly, and monthly temperature and precipitation values are available for analysis, covering the period from 1870-1956. Most of the data are in machine-processable form.

2. BASIC DATA

STATION SELECTION AND QUALITY

Woodstock, Md., is located about 16 miles west-northwest of Baltimore at $39^{\circ}20' N.$, $76^{\circ}53' W.$ Climatological records have been maintained there virtually without interruption since 1870. The length and quality of these records, evident stability of station location in the more recent years, apparent freedom from environmental influence and change, and good prospects of future record continuity, have qualified Woodstock as a member of the Weather Bureau's Climatological Benchmark Network.

Records since 1893 of monthly mean temperature and total precipitation for Woodstock have been subjected to a rigorous analysis of homogeneity. The analysis has revealed two discontinuities in temperature which were evidently associated with undocumented station moves about March 1901 and about January 1914. Between 1901 and 1914, mean temperatures were registering about $2.5^{\circ} F.$ too high in winter and about $1.5^{\circ} F.$ too high in summer, relative to the record since 1914. The record prior to 1901 was approximately homogeneous with the record since 1914. The analysis also indicates that the precipitation record is homogeneous, with the possible exception of a period of several years between 1930 and 1940, when the gage catch at Woodstock was apparently deficient by about 6 percent.

Since the nature of the temperature inhomogeneity is rather confidently known, the relative imprecision of the spectrum analysis (at long wavelengths) can be estimated (see section 4). Other inhomogeneities which may exist in the temperature and precipitation series are evidently so small that, in the writers' opinion, they are incapable of affecting the spectra based on those series to a significant degree.

DATA PROCESSING

The existence of preferred group periods over as long a time interval as possible in both precipitation and tem-

perature records was chosen for investigation. Monthly as well as daily values were selected for analysis. Early in the investigation it was decided that lag periods of more than 100 would be used. The maximum lag period of the then available IBM 650 Bell Laboratory spectrum analysis routine was 100. Therefore, plans were made to make a new program. Actually, it was necessary to make two new programs due to the relatively small magnetic memory drums of the IBM 650, 2,000 words. One of the programs developed will handle lag periods from 0 to 400 while the other will handle lag periods from 400 to 750.

Experience gained in processing data indicated that memory dump routines would be required. Therefore, such routines were made for use during either the input of data or output of information. The memory was "dumped" every hour. This took only about two minutes, but always provided a new starting point if any malfunction occurred in equipment or power. Such a procedure also bypassed the necessity of remaining with the problem until all computation was finished. In other words, work on this project could be curtailed for priority work or stopped at the end of a shift and started again the next day.

This is important, for with large amounts of input data and large lag, input and output rates may be reduced to two cards per minute.

DATA CARD DECKS

The input data to the machines were in punched card form. The data required for this particular study were daily and monthly values of the average temperatures and total precipitation. The average temperatures were measured in $^{\circ} F.$ and the total precipitation in inches.

Daily data, 1895-1956.—Daily values of maximum, average, and minimum temperatures and precipitation were sought. Although inferences indicated the existence of daily data back to 1875, investigation failed to find them. Prior to 1910 there were breaks in the record, usually in the summertime. It was necessary to devise methods of interpolation of the missing observations. Even with these, it was deemed inappropriate to fill in the gap periods prior to 1895 because these gaps began to exceed two months.

To obtain the values to be substituted for missing data, comparisons with nearby stations' observations were made. In general, the curves of daily values of the elements were constructed by standard methods of interpolation and the required values were obtained.

Monthly data, 1875-1956.—Monthly averages of maximum, average, and minimum temperatures and of precipitation could be interpolated much more easily, and for this reason the period could be extended back to 1875. In a test group of cases where monthly values

were computed and compared with observed values, the differences between observed and interpolated were all less than $\pm 0.5^\circ$ F.; generally, their magnitude was near $\pm 0.1^\circ$ F. Thus, there is reasonable assurance that the interpolated values used should not be far from the values which would have been recorded. The monthly precipitation totals were transformed by the equation

$$Y = \sqrt{X} + \sqrt{X+1} \quad (1)$$

where X is the monthly total precipitation [14].

3. PROCEDURES

Tukey's paper [35] which deals with the sampling theory of power spectrum analysis is used as the basis of this study. His subsequent papers as co-author with Blackman [8] have also been utilized. The formulas given here, in essence, are those of Tukey with the following exceptions:

a. The total overall mean is used to determine all serial products rather than the mean for the individual lag interval (see Panofsky and Brier [23]).

b. Division of all covariances by the initial covariance permits the immediate computation of normalized line powers. However, when the data are plotted in this form, the variance at zero lag is not readily available. The formulas used are as follows:

(i) Serial Products

$$SP = \sum_{i=1}^{n-p} (X_i - \bar{X})(X_{i+p} - \bar{X}); \quad (2)$$

$$\bar{X} = (1/n) \sum_{i=1}^n X_i \quad (3)$$

where p is the lag and n is the number of observations in the entire sample.

(ii) Mean Serial Product or Covariances

$$R = SP/(n-p) \quad (4)$$

(iii) Covariance Ratio

$$R/R_0 = R'; R_p/R_0 = R'_p; 0 < p < m \quad (5)$$

(iv) Line Powers

$$L_0 = (1/2m)(R'_0 + R'_m) + (1/m) \sum_{p=1}^{m-1} R'_p \quad (6)$$

$$L_h = (1/m)R'_0 + (2/m) \sum_{p=1}^{m-1} R'_p \cos p h \pi / m + (1/m)R'_m \cos h \pi \quad (7)$$

$$L_m = (1/2m)(R'_0 + (-1)^m R'_m) + (1/m) \sum_{p=1}^{m-1} (-1)^p R'_p \quad (8)$$

where m is the maximum lag.

$$0 \leq p \leq m-1$$

$$0 \leq h \leq m-1$$

(v) Smoothing Formulas (after Hamming and Tukey [15])

$$U_0 = 0.54L_0 + 0.46L_1 \quad (9)$$

$$U_k = 0.54L_k + 0.23(L_{k-1} + L_{k+1}); 0 < k < m \quad (10)$$

$$U_m = 0.54L_m + 0.46L_{m-1} \quad (11)$$

4. RESULTS

SHORT-PERIOD VARIATIONS

For the precipitation fluctuations in the range from 2 days to a month analyses were run for several 5-year intervals. Results for four of these intervals, namely 1910-14, 1941-45, 1946-50, and 1951-55 are shown in figure 1. For the interval 1951-55 the daily temperature data were also analyzed. Their power spectra are depicted in figure 2. The first impression one gets from the curves is one of variety. None of the periods is outstanding. Each of the chosen intervals seems to have its own spectrum. Nonetheless there are some interesting features. In figure 1 these are represented by three lines. The first is around 3 days, the second between 5 and 7 days, and the third in the 15- to 25-day span. In the temperature curves (fig. 2), which represent only one 6-week season, the winter, the 5- to 7-day periods are quite pronounced. There is also some power in the longer intervals over 20 days. Even though these periods are not distinctly fixed and vary markedly from year to year they undoubtedly reflect a physical mechanism in the atmosphere.

The 3-day period is apparently associated with a fast-moving wave around the globe. There are hints of that type of phenomenon in the literature. Portig [25] has called attention to such a rapid pressure wave. Very little is known about the nature of this wave. Angell [2] noted a peak in variance when he analyzed wind fluctuations at the 300-mb. level for an interval of 50 hours. This is a little shorter than the 3-day peak at Woodstock but Angell's data were primarily from the Pacific area.

The 5- to 7-day interval prominence in the power spectrum confirms age-old meteorological knowledge. It may be useful to cite some of the classical findings here. Arctowski [3] in the results of the Belgica Expedition (1897-99) mentioned that the pressure waves near the Antarctic had an average duration of 5 days 6 hours. Meinardus [20] from the data of the German South Polar Expedition (1901-03) noted an average length of pressure waves of 5 days 2 hours. For the data of the Scott Expedition, Simpson [30] calculated the following periods for the Southern Hemisphere and the Antarctic coast:

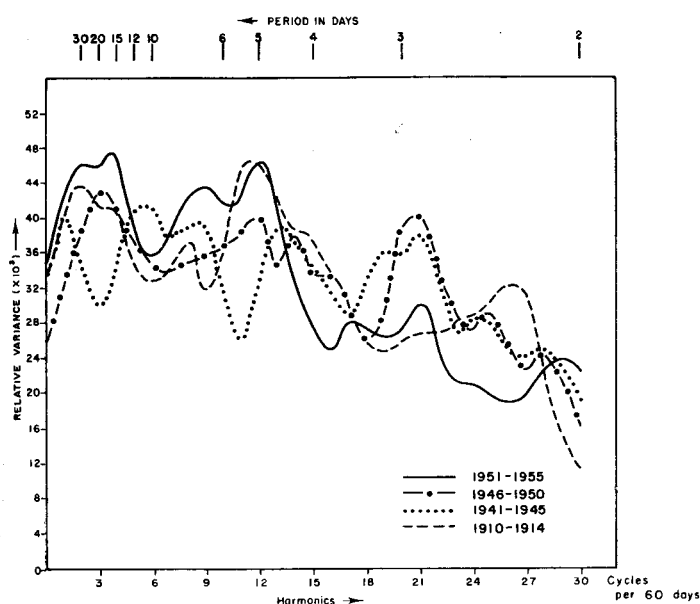


FIGURE 1.—Power spectrum analysis of daily precipitation data, Woodstock College, Md. Maximum lag is 30. The sum of the 30 relative variances equals 1 or 100 percent.

Latitude	1st	2d	3d	4th	wave
52° S.-----	3. 65	7. 32	13. 48	29. 5	days
70° S.-----	3. 39	6. 06	13. 43	43. 1	days

In the Northern Hemisphere A. Wegener [37] found for East Greenland for the year, less the summer months, an average duration of pressure waves of 5.6 days. In the summer the pressure fluctuation lengthened to a 9.4-day period. The 5.6-day pressure wave was also noted by Merecki [21, 22] in Warsaw. In Western Europe L. Descroix [13] described a 5-day period for Paris, and Scoles [28] found the same periodicity in the data for Malta during April. In the August to September season that Mediterranean island showed an 8.2-day fluctuation. H. H. Clayton [11] also propounds 3.5- to 5-day pressure periodicities. In a very careful harmonic analysis A. Defant [12] finds the following waves in precipitation data in the middle latitudes of the two hemispheres:

Latitude	1st	2d	3d	4th	wave
35° S.-----	7. 2	12. 1	16. 6	31. 2	days
45° N.-----	5. 7	8. 7	12. 7	24-25	days

The physical explanation of the 5- to 7-day periodicity pattern seems to be closely related to the long waves in the westerlies, first described by Rossby. In a 4-wave system with a movement of 18° longitude per day one revolution of the whole wave train would take, for example, 20 days so that a fixed point on the surface would experience a wave passage once every 5 days. In a 3-wave system one passage per 7 days would be noted (cf., in

this connection, Saltzman [27]). In the subtropical easterly currents similar wavelengths seem to be prevalent. Hubert [17] noted that the lag correlation for the thickness of the 1000- to 700-mb. layer had a 5-day maximum of 0.7 at San Juan, Puerto Rico.

We may also recall here the rediscovery of the 7-day cycle in connection with the controversial periodic cloud seeding experiments (Langmuir [19], Brier [9]).

The importance such periodicities may have for extended forecasting by numerical procedures has been stressed by Scorer [29]. The unfortunate circumstance is the rather flexible lengths of rhythms—and, as our spectra show, the rather wide variations from year to year. Little is known yet as to what elements determine the wave number in the atmospheric currents. There are some relations to the large oceanic heat sources in winter but what causes changes from a 3-wave to a 4- or more wave system is still a mystery.

The seasonal variability found by others for the short-periodic rhythms made it appear worthwhile to try power spectrum analysis for a few years by seasons. In doing this we adopted the meteorological seasons advocated by Baur [4,5] for the middle latitudes of the Northern Hemisphere. He adduces important arguments that the chosen intervals are in their circulation patterns more homogeneous than the customary 3-month seasons. His intervals are 45 or 46 days long and comprise the following dates:

- Season 1: January 1–February 14 (High winter)
- 2: February 15–March 31 (Pre-spring)
- 3: April 1–May 16 (Full spring)
- 4: May 17–June 30 (Pre-summer)
- 5: July 1–August 15 (High summer)
- 6: August 16–September 30 (Early autumn)
- 7: October 1–November 15 (Main autumn)
- 8: November 16–December 31 (Pre-winter)

The graphs of figure 3 show the power spectra for the daily precipitation at Woodstock College, Md., for each of the years 1951–55. At first glance these show little consistency. Although the non-normal distribution of the precipitation data raises doubts as to their relevancy, the 5 and 95 percent confidence lines corresponding to a white spectrum* are indicated on these diagrams. From a statistical viewpoint alone, in spite of the fact that there are values which exceed these limits, their number does not exceed what one would have to expect for the number of spectral lines shown here. There are 30 spectral lines to the individual season. In 5 years, i.e., five winter seasons, one would expect 5 percent of the 150 spectral lines or about 8 cases before one could consider particular periods as significant. In other words, about 8 significant peaks

*A "white spectrum" is a random spectrum, in which the expected value of the power in each harmonic is the same.

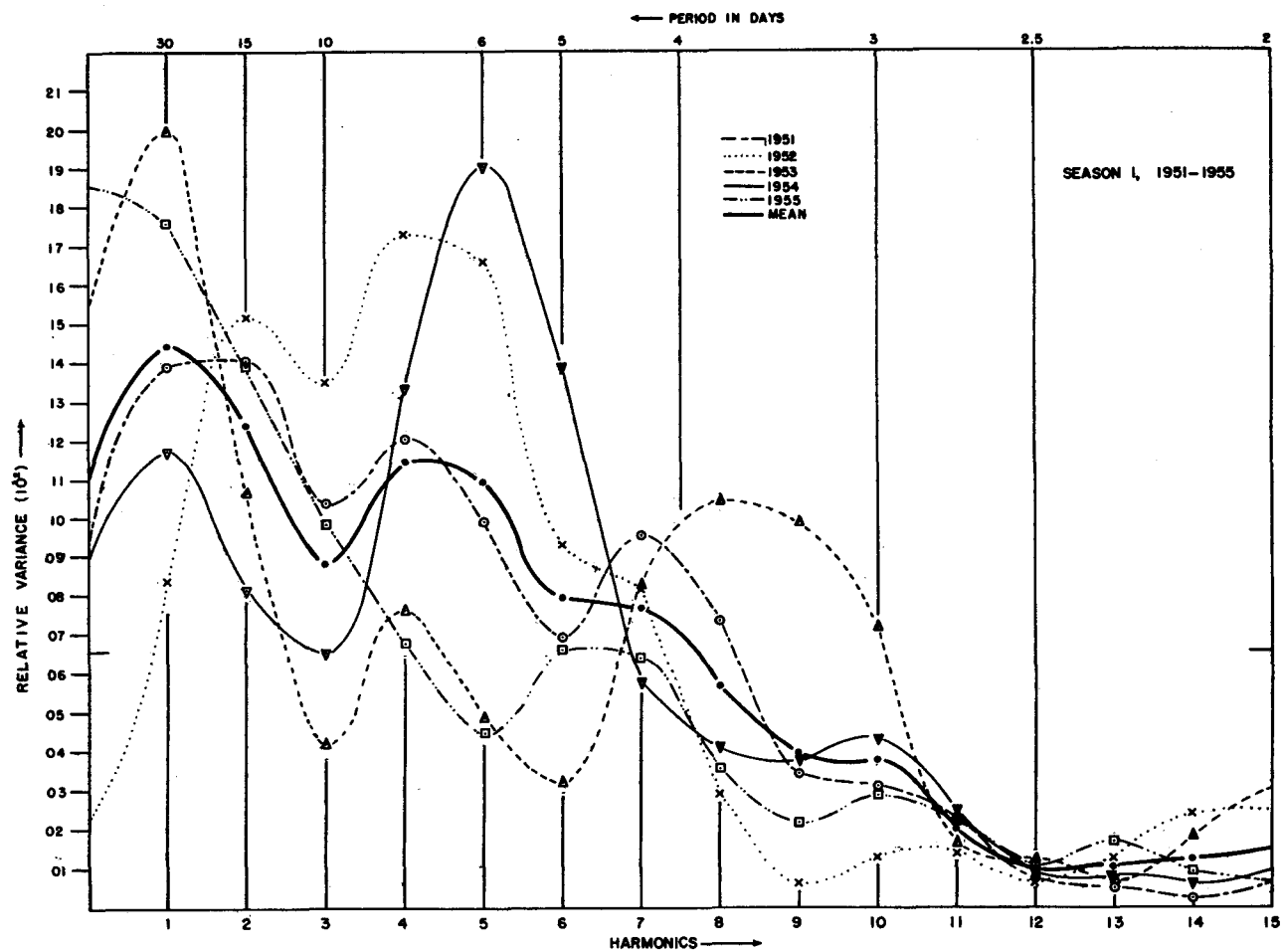


FIGURE 2.—Power spectrum analysis of daily average temperature data (°F.), Woodstock College, Md., for Season 1 (Jan. 1–Feb 15) 1951–55. Maximum lag is 30. The abscissa values at the base of the chart are linear in terms of harmonics; those at the top are in days. The average spectral (white) value is shown by the horizontal tick marks at 0.066. The heavy solid line connects the tips of the individually averaged spectral lines. All spectra are smoothed by the Hamming procedure.

could occur by chance. No season exhibits even that many significant spectral lines. Also, with respect to consecutive seasons through the years there is an insufficient number of significant spectral lines. For example, if the period of 10 to 15 days is selected, out of 200 spectral lines about 10 can be expected to show significance by chance. In our case we count actually 9. In the 2.5- to 3.5-day period 8 would be expected by chance, however, none shows up; yet there is a tendency for peaks at these periods in many of the single seasonal spectra. Similar conditions prevail with respect to other periods. Nor do the minima show statistical significance.

It might, nevertheless, be worthwhile to linger a little yet in the discussion of these data. They show definitely that in almost all seasons considerable power resides in relatively long periods. There is no tendency, however, for the spectra in these 5 years to show any particular preferences for certain periods by seasons. If one can attribute any meaning to these patterns it is that most of the seasons had their own characteristic "signature". It

is premature to say whether such spectra could be used as circulation labels for the season. The only startling case of similarity of pattern is for the high winters (season 1) of 1954 and 1955. These were not particularly similar in their characteristics but both had rather well-marked 4-trough systems for most of the season over the higher latitudes of the Northern Hemisphere. This could explain the high power in the 5- to 6-day rhythm.

Perhaps another noteworthy element is the relative similarity shown for the seasons 3–1952, 8–1952, 1–1953, 2–1953, 6–1953, 7–1953, 2–1955 and 3–1955. All of these show their main power in the 10- to 15-day spectral band with a monotonous decline to both the longer and shorter periods. These may be related to the so-called index cycles. Takahashi and collaborators [33] referred to standing waves in their harmonic analysis of the 500-mb. heights in the Northern Hemisphere. However, their intervals were much longer than those noted here. But oscillations of the general circulation, as already noted in the introduction, can be derived even from elementary

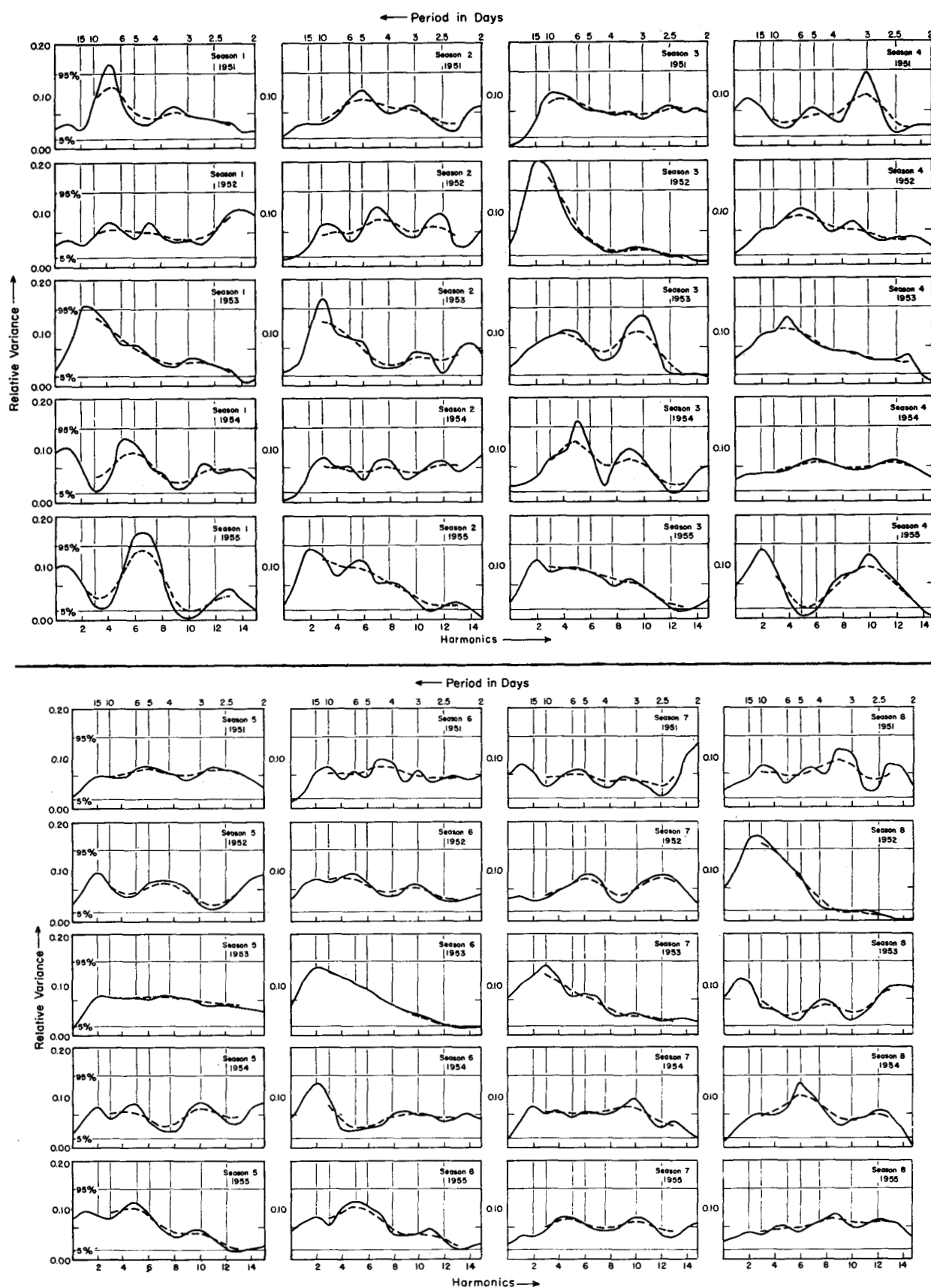


FIGURE 3.—Power spectrum analysis of Woodstock College daily precipitation data by 6-week seasons, January 1, 1951, through December 31, 1955. The abscissa values at the base of the chart are linear in terms of harmonics; those at the top in days. The χ^2 5 and 95 percent levels of significance relative to a white spectrum are shown. The average spectral (white) value is shown by the horizontal tick marks at 0.066. The solid line was produced by the Hamming smoothing procedures of the line power. The dashed lines are a binomial smoothing to the 4th power.

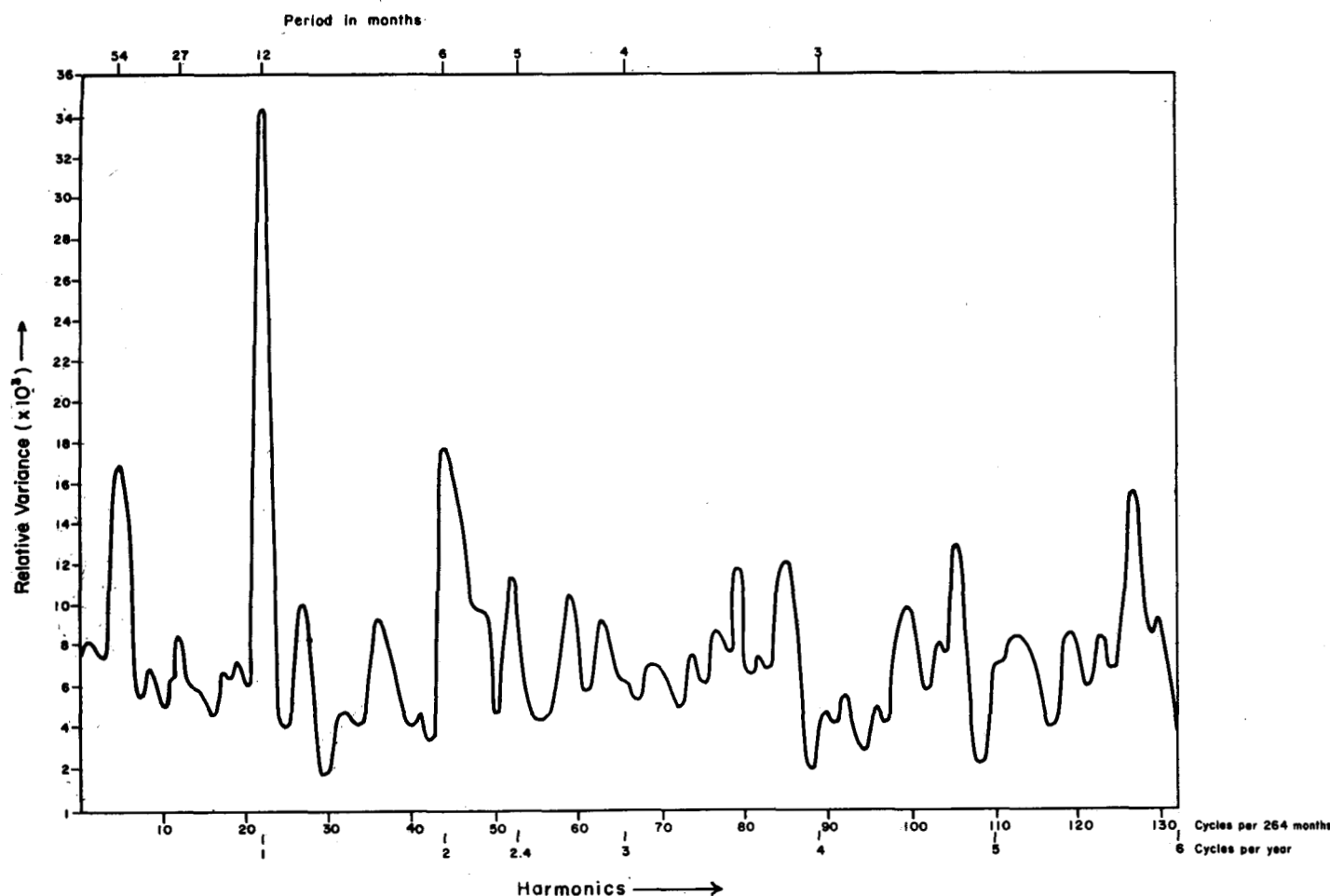


FIGURE 4.—Power spectrum analysis of monthly total precipitation, Woodstock College, Md., 1895-1956. 743 months, maximum lag 132, transformed by equation (1). The sum of the 132 smoothed relative variances equals 1 or 100 percent.

theoretical models. The absolute values calculated from these presently give only an order of magnitude. Smagorinsky [31] found an 11-day period from his calculations. The importance of this does not lie in the exact value, which will undoubtedly change as more sophisticated models are used, but rather in the fact that oscillatory elements result from theoretical approaches of the same order of magnitude as those noted here from the empirical end. This suggests that this avenue of approach, while presently barren of practical results, should not yet be considered as quite closed. It is, of course, obvious that a single-station analysis of a most variable element, such as precipitation, permits only a very limited outlook on these broad-scale phenomena in the atmosphere. To the man in the street the variation of precipitation and its rhythmical tendencies might be of paramount importance. He may be concerned if his favorite outdoor sport is rained out two or more weekends in succession. For the meteorologist, however, other indices are likely to be much better suited and more general in nature for spectral analysis.

LONG-PERIOD VARIATIONS

For purposes of studying long-period (low frequency) variations in climatological series, optimum techniques of power spectrum analysis are characterized by the following:

1. At least in the case of temperature, the monthly data are advantageously pre-whitened* to remove the annual cycle, whose large contribution to the total variance of such data may otherwise impair the ability of the analysis to resolve details at the other frequencies in which our interest primarily lies.

2. Large maximum lags must be used to resolve adequately fluctuations of very low frequency. Moderately small maximum lags (≤ 100 months) may be justified if we anticipate the *population* spectra to be quite monotonous at the low-frequency end. Recalling the discussion in section 1, however, we may be no less justified in anticipating an irregular power distribution in the population spectra, of which the calculated spectra are small samples.

*See for example, Blackman and Tukey [8] for a discussion of the meaning of pre-whitening.

Therefore, spectral analysis involving a maximum lag of 546 months (45.5 years) was employed in which the resolution is sufficient to distinguish between fluctuations of the order of 10 years in period (the 11-year sunspot cycle) and those of the order of 30 years in period (the double sunspot and Brückner cycles), and between the latter and secular trend.

The use of such large maximum lags with a record only 86 years long, however, introduces very large amplitudes of spurious spectral power at a few randomly distributed wavelengths. In this manner, it complicates the testing of amplitude significance in the usual circumstance where certain *specific* wavelengths are not anticipated in advance to be significant by a priori physical hypothesis. That is, assuming a 95 percent significance level, a spectrum with a maximum lag of 546 months would be expected to contain between 17 and 37 "significant" peaks even if the spectrum were that of a random series. The use of a higher significance level to avoid Type I errors of this kind obviously increases the probability of overlooking physically real fluctuations which may in fact be present.

Precipitation spectra.—In this section, we shall be concerned primarily with the 546-month maximum lag analysis of precipitation, in which the seasonal march was first removed from the data. First, however, let us refer to a 132-month maximum lag analysis in which the seasonal march has been retained. This spectrum, shown as figure 4, applies to monthly precipitation totals at Woodstock from 1895 to 1956, which were transformed by equation (1) [14] which approximately normalizes an incomplete gamma-distributed variable (X) such as monthly total precipitation (Thom [34]). The annual cycle appearing in this spectrum is the most prominent feature of it, and accounts for 3.4 percent of the total variance of monthly precipitation. Its amplitude can be tested by the sampling theory of Tukey [35]. According to Tukey, the ratio of any spectral ordinate to the local ordinate of the *smooth* spectrum (here assumed equal to $1/132$, the reciprocal of the maximum lag) is distributed as chi-squared/degrees of freedom. The degrees of freedom in turn are given by

$$\text{d.f.} = (2N - m/2)/m, \quad (12)$$

where N is the total record length, and m is the maximum lag used. In this case, $N=743$ months, $m=132$ months, and the degrees of freedom associated with a spectral peak involving k harmonics is $10.8k$. For the annual cycle, $k=1$, and the 99.9 percent confidence limit of the spectral amplitude (relative variance) is 0.022. The relative variance actually associated with the annual cycle is 0.034, which is thus highly significant.

Two other power maxima in this spectrum are also worth comment. The peak near a period of 6 months, while considerably less pronounced than that of the annual period, is significant at the 99 percent confidence

level. Its physical reality is hardly to be doubted, inasmuch as the second harmonic of the annual period is frequently required along with the first harmonic to account for the total variance of the seasonal march in climatological data.

The nearly equally pronounced maximum near a period of 53 months (4–5 years) is of unique interest because, if real, its interpretation must be essentially "meteorological" rather than "astronomical." This spectral peak falls just short of significance at the 99 percent level. At least one peak of such magnitude should be expected to occur by chance somewhere in a spectrum containing as many as 132 harmonics. On the other hand, it should be recalled from the discussion in section 1 that quasi-cyclical variation of climate with periods of several years has often been suspected before. The nature of this apparent cycle is further investigated below.

Monthly total precipitation in the full 87-year record at Woodstock was subjected to a spectrum analysis with a maximum lag of 546 months. This spectrum is shown in figure 5 by the lighter line. The heavier line was produced by drawing a line through points obtained by averaging consecutive groups of 10 harmonics. For this analysis, the precipitation data were first transformed by equation (1), and then expressed in terms of departure from their monthly average by the relation

$$Z = Y - \bar{Y} + \bar{\bar{Y}} \quad (13)$$

where \bar{Y} is the average of Y in that particular calendar month, and $\bar{\bar{Y}}$ is the grand average of Y in all calendar months. By this means, the variance due to the seasonal march, i.e., to all six resolvable harmonics of the annual cycle, has been completely eliminated from the spectrum; only the meteorological fluctuations remain.

The large resolution of this spectrum enables an examination of the single and double sunspot cycles, the Brückner cycle, and the higher harmonics of the double sunspot cycle which Abbot [1] claims to be useful in long-range forecasting. The extreme irregularity of the spectrum cannot, of course, be taken to mean that the true population spectrum of monthly precipitation—however we may care to define it—is comparably irregular. As with all work in spectrum analysis, an attempt is made to obtain an estimate of the true power spectrum of an infinitely long record from a finite portion of this record. This particular part of the record being analyzed may be considered as one of infinitely many pieces of record of similar length which could be obtained. The power spectrum constructed from a sample piece of record is subject to sampling variation within the period. Also the spectrum obtained may be considered in total as being a sample of the true spectrum. This means that a more definitive answer would be obtained by examining a sequence or series

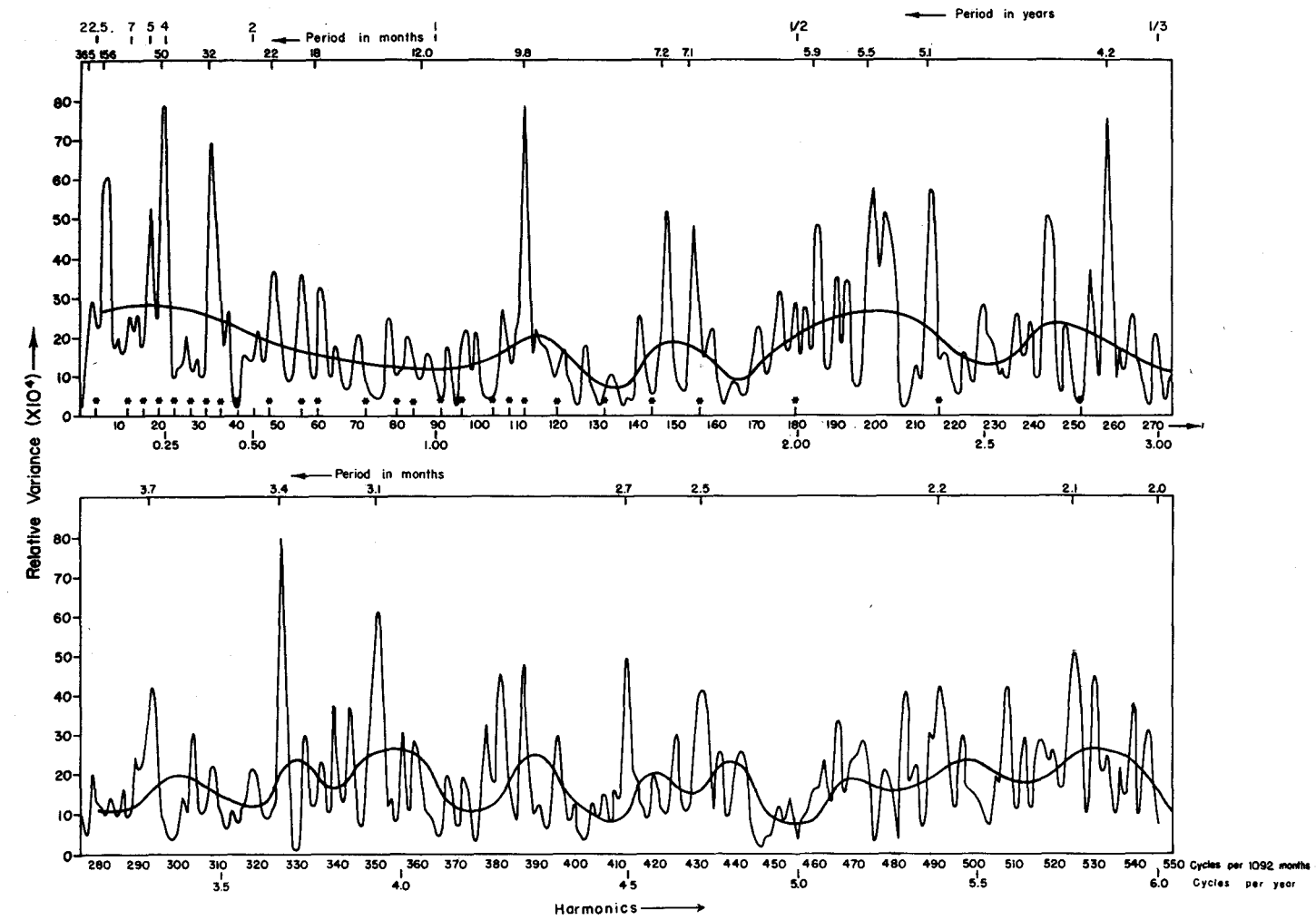


FIGURE 5.—Residual normalized power spectrum analysis of monthly total precipitation, Woodstock College, Md., 1044 months, 1870-1956. Maximum lag 546. Data values were transformed by equation (1) and modified by equation (13). The thin line connects the tips of the Hamming-smoothed line powers. The sum of the 546 line powers equals 1 or 100 percent. The heavy line connects the averaged line powers for each consecutive group of ten. The average spectral (white) is $1.000/546$ or 18.31×10^{-4} . The 5 and 95 percent bounds for this average are 2.4×10^{-4} and 45.8×10^{-4} . Highest harmonics are probably inflated somewhat by aliasing. "Stars" mark periodicities after Abbot.

of spectra of finite and equal records. In this particular spectrum, only 3.3 degrees of freedom are associated with a power peak contributed by one harmonic. The critical ordinate magnitudes in figure 5 which may be assigned various levels of significance, assuming the null hypothesis of a white (rectangular) spectrum, are listed in table 1. Inspection of the data plotted in figure 5 reveals that the number of harmonics formally assigned significance on this basis is not itself significantly different from the number of harmonics expected to reach each significance level by chance if the precipitation series were actually random.

The power maximum in figure 4 which lies between 4 and 5 years is resolved in figure 5 into a principal maximum between 4.1 and 4.4 years (harmonics 21 and 22), and a secondary maximum between 4.7 and 5.2 years (harmonics 18 and 19). In relation to the null hypothesis of white noise, these peaks are significant at the 99.9 per-

cent and 95 percent levels, respectively. A third maximum between 12.1 and 16.6 years (harmonics 6 and 7) also appears, which is significant at the 99 percent level according to the same null hypothesis.

In place of assuming white noise as in table 1, suppose we stipulate that the population spectrum is best repre-

TABLE 1.—Significance of peaks in 546-month maximum lag spectra (figs. 5 and 7) based on Tukey's sampling theory with null hypothesis that population spectrum is that of white noise

Number of harmonics contributing to peak, and degrees of freedom	Significance level (percent)		
	95	99	99.9
1 3.3 d.f.	0.00467	0.00669	0.00950
2 6.6	.00370	.00496	.00652
3 9.9	.00336	.00427	.00544
4 13.2	.00316	.00387	.00490

Mean variance of white spectrum = $1/546 = 0.00183$

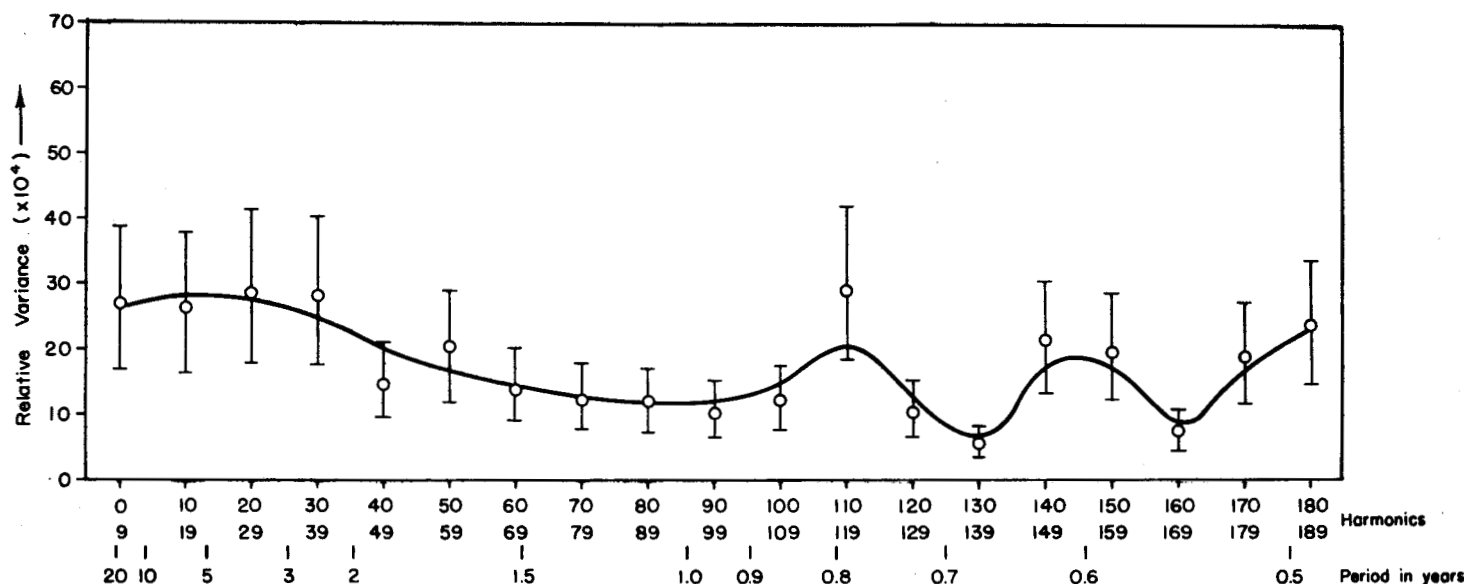


FIGURE 6.—Residual normalized power spectrum analysis of monthly total precipitation, Woodstock College, Md., 1044 months, 1870–1956. The harmonics have been averaged by consecutive groups of ten from figure 5. Averages are shown by circles and the 5 and 95 percent χ^2 limits by the extent of the vertical lines.

sented by the broad, modestly inflated power maximum involving the lowest 40 harmonics, which appears in figure 6. As indicated above, averages for consecutive groups of 10 harmonics were plotted and a line drawn through them to produce the heavier curve of figure 5. These same averages are indicated in figure 6 by the circles. Chi-squared limits of 5 and 95 percent are indicated by the horizontal tick marks on the ends of the vertical lines drawn through the circles. The heavier line of figure 6 has been produced by establishing 30 and 70 percent chi-squared limits and then drawing the curve subjectively with these limits as a guide. In such a case, only the two sharp peaks at harmonics 21 and 22 and harmonics 6 and 7 are significant at the 95 percent level, which is not impressive. Hence, one may interpret the precipitation spectrum in one of two ways: either the population spectrum at these long wavelengths is irregular with certain rather narrow peaks, or the population spectrum is rather modestly inflated in the entire interval between the zeroth and, say the 40th harmonics. With the present lack of corroborating evidence to suggest the reality of narrow bands of spectral power in this region of the spectrum, the writers tentatively prefer to consider the population as given by the smooth curves in figures 5 and 6, in which case *none* of the sharp spectral peaks corresponding to periods longer than 2 years is probably significant per se.

Temperature spectrum.—The 546-month maximum lag analysis of monthly mean temperature at Woodstock in the period 1870–1956 is shown as figure 7. In preparing this spectrum, equation (3) was first applied to the data

in order to remove the seasonal march, described by the harmonics of the annual cycle. This had the effect of reducing the total variance from $240 (^\circ\text{F.})^2$ to $11 (^\circ\text{F.})^2$ and shows that 95 percent of the variance is accounted for by the seasonal march.

It will be noticed immediately that the first few harmonics are remarkably inflated in magnitude. Recalling that the observed temperatures between 1901 and 1914 were significantly biased (see section 2), one could anticipate that those particular harmonics would be inflated to some extent. Since the magnitude of the slippage of mean temperature could be rather confidently determined, it is possible to estimate the effect of the data inhomogeneity on the spectrum. One approach is to apply the Fourier integral theorem to the function illustrated in figure 8, which corresponds to the shape of the inhomogeneity in the Woodstock data. This leads directly to an expression for the amplitude of the sine and cosine components of the function, whose fundamental period is 2π .

Since the power contributed to the spectrum by a given harmonic is equal to half the square of the amplitude of that harmonic, the spectral power in the i th harmonic (whose period is $2\pi/i$) is found to be

$$A_i^2 = \frac{.292}{i^2} [1 - \cos(.3i\pi)], i = 1, 2, \dots \quad (14)$$

where A_i^2 is in $(^\circ\text{F.})^2$. Expressing this as a fraction of the total variance of monthly mean temperature at Woodstock, taken as $11 (^\circ\text{F.})^2$, and suitably relating the fundamental periods of the function and the data

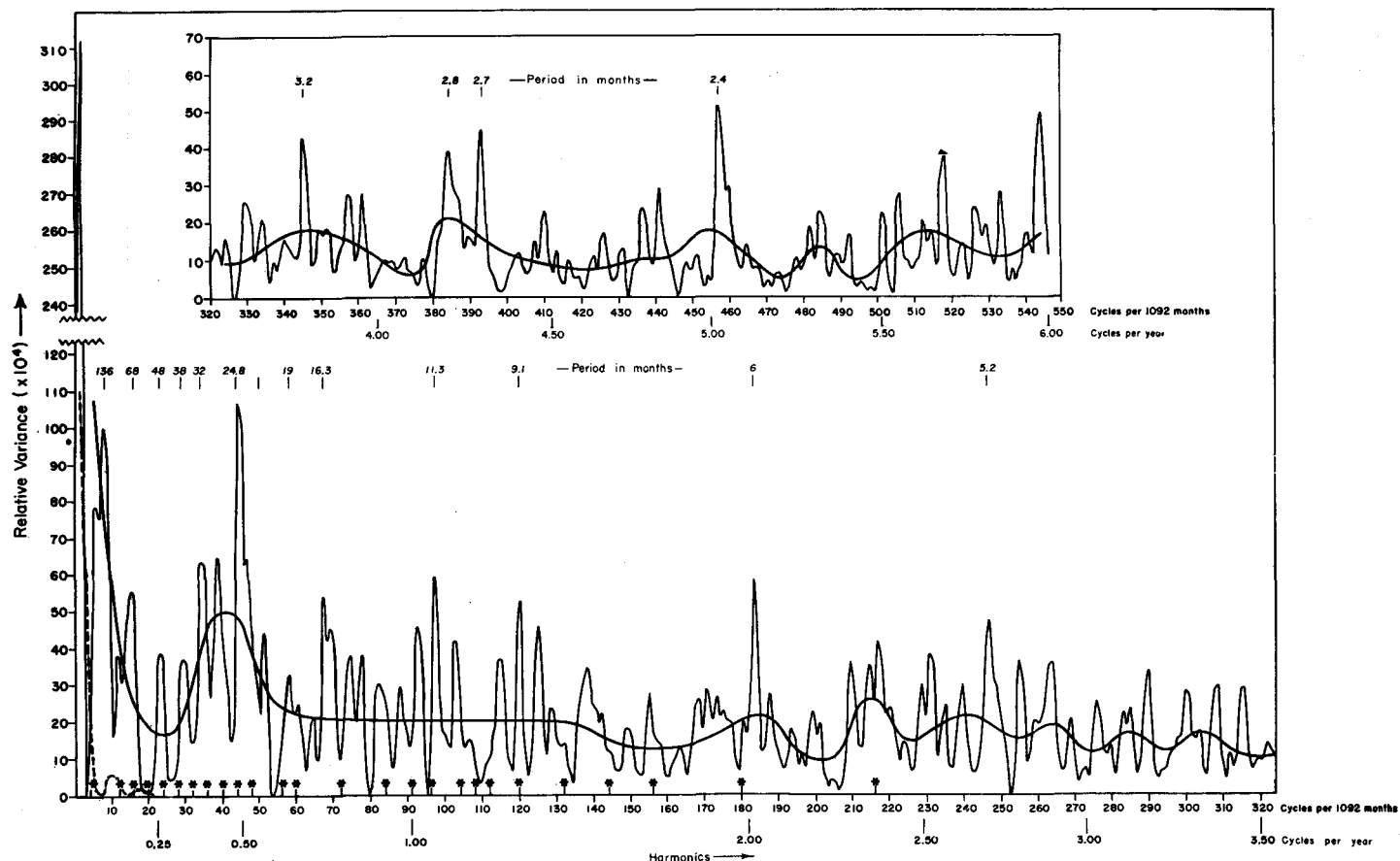


FIGURE 7.—Residual normalized spectrum analysis of monthly average temperature, Woodstock College, Md., 1032 months, 1870–1956. Maximum lag 546. Monthly means were extracted from data prior to analysis. The sum of the 546 ordinates equals 1.0. The thin line is the Hamming-smoothed line spectra. The heavy line connects the averaged line powers for each consecutive group of ten. The average spectra (white) is $1.000/546$ or 18.31×10^{-4} . The 5 and 95 percent bounds for this average are 2.4×10^{-4} and 45.8×10^{-4} . The dashed curve is the contribution to variance by the temperature inhomogeneity of 1910–14. Highest harmonics are probably inflated somewhat by aliasing. “Stars” mark periodicities after Abbot.

series itself, one can estimate the relative uncertainty of the normalized spectrum in figure 7 introduced by the data inhomogeneity. The spectrum due to the inhomogeneity alone has been indicated in figure 7, and may be considered as the approximate *maximum* error at corresponding frequencies of the main spectrum which possibly has been caused by the inhomogeneity.

The significance of the peaks in figure 7, stipulating a null hypothesis of white noise, can be estimated by the aid of table 1. Details of the spectrum in the region of the first 25 harmonics, corresponding to all wavelengths longer than about 3.6 years, are shown in table 2. Even after the most pessimistic allowance for the inhomogeneity in the data, one sees that the amplitude of fluctuations longer than a half-century in period is highly significant. Although the Brückner cycle and the double (22-year) sunspot cycle are missing, considerable power is present in harmonics 6 through 9, corresponding to periods between 9.6 and 16.6 years. The harmonic which

contains the single (11-year) sunspot cycle achieves significance at the 99.9 percent level. A lesser peak is found in harmonics 15 and 16, the latter of which contains the second harmonic of the 11-year sunspot cycle, believed by some authors to have physical importance [4]. The significance of harmonic 16 is 95 percent, while that of harmonics 15 and 16 jointly is 99 percent.

Another very prominent peak in the spectrum of figure 7 is found for harmonics 44 and 45, corresponding to periods of 24.0 to 25.1 months. The power in each of these harmonics is significant at the 99.9 percent level, again assuming the null hypothesis of the white spectrum. No physical interpretation of the sharp spectral peak near 25 months suggests itself.

As in the case of the precipitation spectrum, the null hypothesis of white noise, which was assumed in ascribing significance levels for these sharp spectral peaks of temperature, can be challenged as unrealistic. A different null hypothesis can be justified by averaging the power in

TABLE 2.—Details of first 25 harmonics of 546-month maximum lag spectra of temperature and precipitation (figs. 5 and 7)

Harmonic number	Inclusive period (years)	TEMPERATURE			PRECIPITATION			Remarks
		Power ¹		Significance ³	Power		Significance ³	
		Unsmoothed	Smoothed ²		Unsmoothed	Smoothed ²		
0	182-∞	-0.0026	(0.0182)	**	0.0000	0.0002		Secular trend.
1	60.8-182	(.0438)	(.0214)	**	.0005	.0008		} Brückner cycle. Double sunspot cycle.
2	36.4-60.8	(.0003)	(.0099)	**	.0022	.0020		
3	26.0-36.4	(-.0050)	(-.0009)	*	.0029	.0028		
4	20.2-26.0	(.0061)	(-.0014)	*	.0033	.0023		
5	16.6-20.2	(-.0018)	(.0042)	*	-.0007	.0022		} Single sunspot cycle.
6	14.0-16.6	(.0146)	(.0075)	*}	.0081	.0058	†}	
7	12.1-14.0	.0023	.0075	*}	.0066	.0060	†}	
8	10.7-12.1	.0123	.0099	**}	.0026	.0031	†}	
9	9.6-10.7	.0119	.0084	*}	.0008	.0018		} Second harmonic of single sunspot cycle.
10	8.7-9.6	.0039	.0016		.0031	.0020		
11	7.9-8.7	.0041	.0024		.0004	.0016		
12	7.3-7.9	.0045	.0038		.0026	.0022		
13	6.7-7.3	.0018	.0030		.0030	.0025		} Second harmonic of single sunspot cycle.
14	6.3-6.7	.0044	.0037	†}	.0011	.0021		
15	5.9-6.3	.0041	.0051	†}	.0036	.0025		
16	5.5-5.9	.0081	.0055		.0014	.0017		
17	5.2-5.5	.0006	.0024		.0007	.0023		} Second harmonic of single sunspot cycle.
18	4.0-5.2	.0006	.0004		.0070	.0053	†}	
19	4.66-4.9	-.0002	.0000		.0059	.0042	†}	
20	4.44-4.66	.0001	.0001		-.0028	.0024		
21	4.24-4.44	.0006	.0004		.0113	.0076	*}	
22	4.05-4.24	.0005	.0021		.0093	.0079	*}	
23	3.87-4.05	.0072	.0039		.0012	.0029		
24	3.71-3.87	-.0004	.0016		.0002	.0009		
25	3.57-3.71	.0007	.0004		.0022	.0012		

¹ Temperature values in parentheses have been whitened to maximum extent permitted by data inhomogeneity.

² Smoothing by Hamming formula (see text, section 3).

³ Null hypothesis corresponds to white noise. †=significance at 95 percent; * at 99 percent; ** at 99.9 percent level.

successive blocks of 10 harmonics each, and fitting a curve to these averages which is as smooth as their fiducial limits of amplitude permit. Such curves for both the temperature and precipitation spectra are shown in figures 6 and 9. The fiducial limits shown are 5 and 95 percent limits, which could be readily calculated by adding the degrees of freedom associated with each harmonic involved in the averages. It will be noticed that the smooth curves in figures 6 and 9 are *not* consistent with the assumption of white (rectangular) population spectra, but that the inconsistency is serious only in the case of temperature. If the temperature curve in figure 9 is then used as the null hypothesis for testing the sharp spectral peaks discussed above, one finds, for example, that the sharp peak at harmonics 44 and 45 (periods near 25 months) is significant at the 99 percent level only, which, although noteworthy, is not overly impressive in a spectrum of such fine resolution and in the absence of an a priori reason for expecting to find power at that precise wavelength. But now the broad spectral maximum in the smoothed spectrum of figure 9 near this same period of 25 months must be recognized as being highly significant in its own right. This significance evidently surpasses the 99.99 percent level. The maximum accounts for roughly one quarter of the variance of *annual* mean temperatures.

With this revised null hypothesis of the temperature spectrum, the sharp spectral peak corresponding to the 11-year sunspot cycle loses significance even at the 95 percent level. This conclusion is readily vitiated, how-

ever, by slight changes in the exact form of the fitted curve in figure 9. The second harmonic of the solar cycle (about 5.6 years in period) also loses significance at the 95 percent level under this revised null hypothesis.

A further use of both the temperature and precipitation spectra is that of verifying Abbot's [1] hypothesis to the effect that selected harmonics of the double sunspot cycle can be used as the basis of long-range weather prediction.

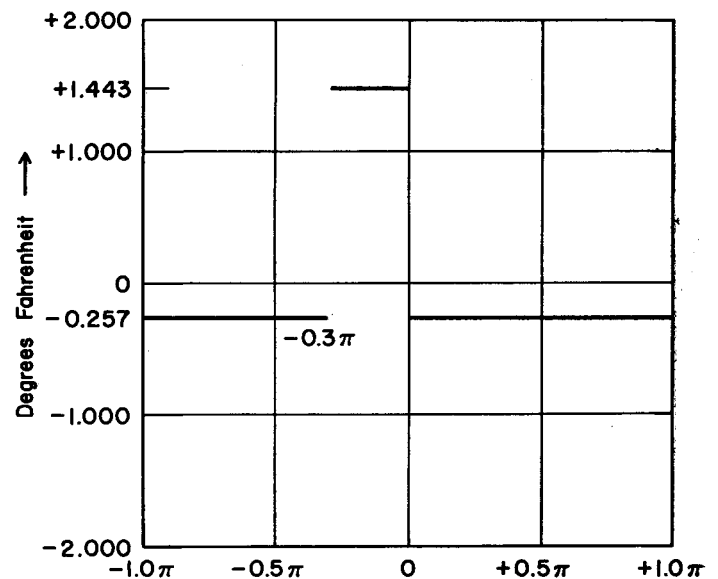


FIGURE 8.—Function resembling data inhomogeneity of monthly average temperature at Woodstock College, Md., in period 1870-1956.

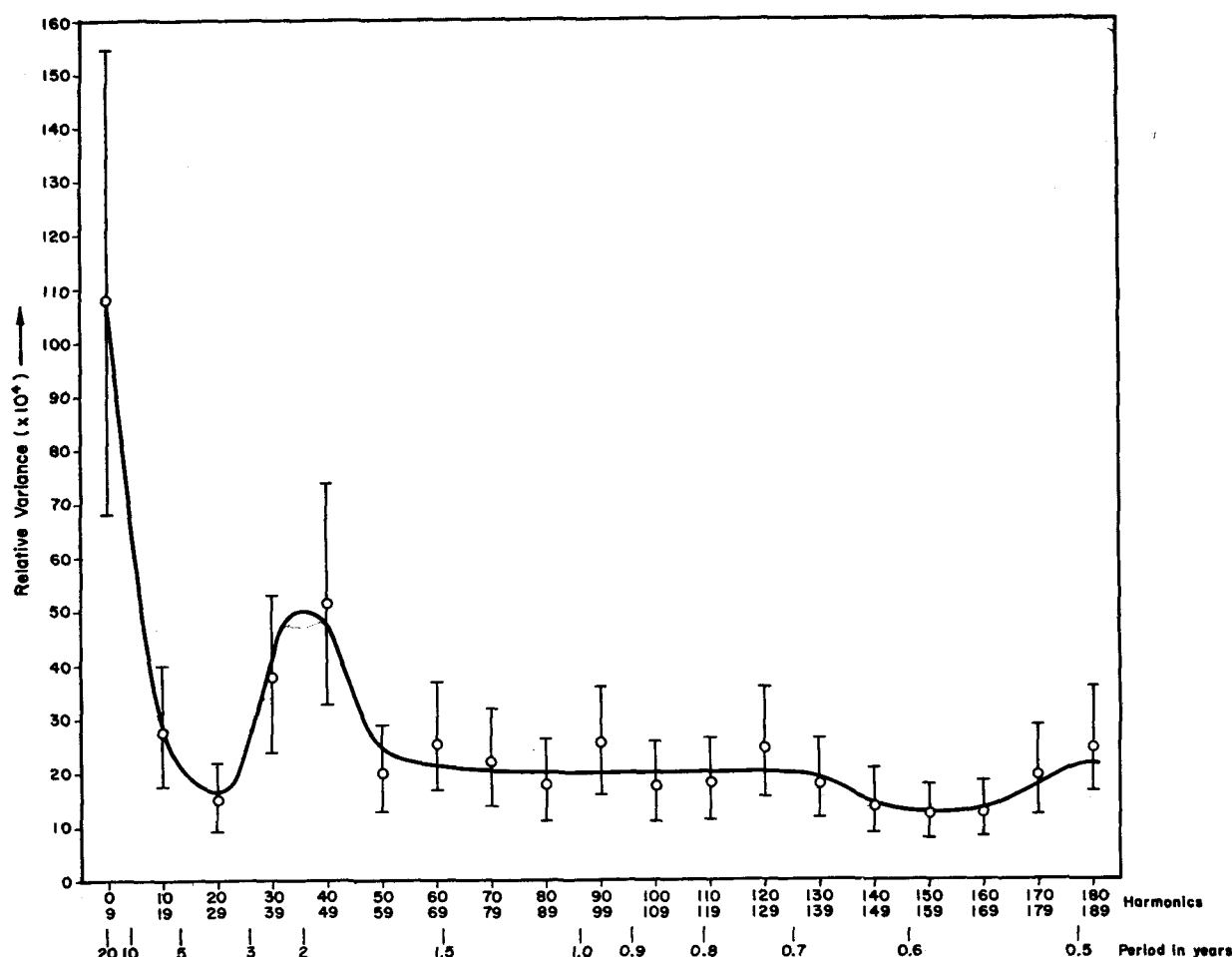


FIGURE 9.—Residual normalized power spectrum analyses of monthly average temperature, Woodstock College, Md., 1032 months, 1870-1956. The harmonics were averaged by consecutive groups of ten from figure 7. Averages are shown by the circles, the 5 and 95 percent χ^2 limits by the extent of the vertical lines.

Each harmonic of the double sunspot cycle corresponds very nearly to each *fourth* harmonic of the fundamental period of the Woodstock analyses discussed in this section. It is, therefore, a simple matter to sum the powers in those harmonics which correspond to the *solar* harmonics selected by Abbot, and to compare this sum with the sum of the powers in the remaining harmonics to determine whether the former can "explain" more of the total variance of temperature and (transformed) precipitation than one could expect from coincidence.

In his study of Arizona precipitation, Abbot used 28 harmonics of the double sunspot cycle, the highest of which corresponds to a period of about 4.34 months. Harmonic 2, the 11-year cycle, was not included. If the population spectra at Woodstock are assumed to be white noise, these 28 harmonics may be expected to account for 5.13 percent of the total variance. In the Woodstock precipitation spectrum, they actually account for 4.97 percent of the total, which is not significantly different from 5.13 percent. In the temperature spectrum, the 28

harmonics account for 7.36 percent of the total, which is significantly different from 5.13 percent at the 99 percent level according to the chi-square test with 92 available degrees of freedom. Since, however, we have had to conclude that the population temperature spectrum cannot in fact be white, we have to take a second look at this last conclusion. Assuming the population spectrum to be the temperature curve in figure 9, we find that Abbot's selected harmonics should be expected to contribute about 7.45 percent of the total variance instead of the 5.13 percent applying to a rectangular spectrum. This compares closely with the 7.36 percent total actually contributed by the selected harmonics. Because the smooth spectrum in figure 9 does not itself exhibit singular power at harmonics corresponding to the solar harmonics, we can safely conclude that no evidence can be found in the Woodstock data to support Abbot's hypothesis. As variations of temperature and precipitation are highly correlated over large geographical areas, this strongly implies that Abbot's method of prediction as applied to any part of

the eastern central United States would show negligible skill over extended periods of time.

Conclusions concerning long-period variations.—The results of the power spectrum analyses of the Woodstock data lead us to the following conclusions, with respect to long-period variations:

1. The spectrum of temperature reveals, in part, two highly significant maxima of variance, one with periods between about 1.8 and 2.7 years, and the other with periods longer than about 50 years.

2. The spectrum of precipitation reveals only minor departures from randomness, the most notable of which consists of a modest inflation of variance in periods longer than about 2 years.

3. The 11-year sunspot cycle is suggested in the temperature spectrum, but not in the precipitation spectrum. Its level of significance in the former is 99.9 percent under the null hypothesis of a white spectrum, but somewhat less than 95 percent under the null hypothesis corresponding to the smooth spectrum shown in figure 9. Its contribution to the total variance of *annual* mean temperatures is about 3 percent.

4. The double (22-year) sunspot cycle is absent from both the temperature and precipitation spectra.

5. The Brückner cycle, of the order of 35 years in period, is virtually absent from both the temperature and precipitation spectra.

6. The second harmonic of the solar cycle, about 5.6 years in period, occurs in the temperature spectrum with a significance of 95 percent under the null hypothesis of a white spectrum, but with a lesser significance under the null hypothesis corresponding to the smooth spectrum in figure 9.

7. In the case of both the precipitation and temperature spectra, major contributions to the total variance of monthly data derive from all portions of the spectra. There are no important gaps in either spectra except in a relative sense, as, for example, the relative minimum in the temperature spectrum near a period of 4 years.

8. The harmonics of the double sunspot cycle, used by Abbot in his scheme of long-range prediction, do not contribute more to the total variance of either temperature or precipitation at Woodstock than the amount ascribable to chance.

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REFERENCES

1. C. G. Abbot, "Detailed Procedure Used in Abbot's Method of Long-Range Weather Forecasting," *Solar Energy*, vol. 2, No. 1, 1958, pp. 26-35.
2. J. K. Angell, "Lagrangian Wind Fluctuations at 300 mb. Derived from Transosonde Data," *Journal of Meteorology*, vol. 15, No. 6, June 1958, pp. 522-530.
3. H. Arctowski, *Résultats du voyage du S. Y. Belgica 1897-1899*, Anvers, 1904, 150 pp.
4. F. Baur, *Physikalisch-Statistische Regeln als Grundlagen für Wetter- und Witterungsvorhersagen*, vol. I, 1956, 137 pp.; vol. II, 1958, 152 pp., Akademische Verlagsgesellschaft, Frankfurt am Main.
5. F. Baur, "Die Meteorologischen Jahreszeiten," *Wetterkarte des Deutschen Wetterdienstes*, Berlin, Beilage No. 143, September 1958.
6. H. P. Berlage, "The Southern Oscillation, a 2-3 Year Fundamental Oscillation of World-Wide Significance," *Scientific Proceedings*, Rome Meeting 1954, International Union of Geodesy and Geophysics, Association of Meteorology, London, 1956, pp. 336-345.
7. H. P. Berlage, "Fluctuations of the General Atmospheric Circulation of More Than One Year, Their Nature and Prognostic Value," *Mededelingen en Verhandelingen*, No. 69, Koninklijk Nederlandsch Meteorologisch Instituut, 'S-Gravenhage, 1957, 152 pp.
8. R. B. Blackman and J. W. Tukey, *The Measurement of Power Spectra from the Point of View of Communications Engineering*, Dover Publications, New York, 1958, 190 pp.
9. G. W. Brier, "7-Day Periodicities in May, 1952," *Bulletin of the American Meteorological Society*, vol. 35, No. 3, Mar. 1954, pp. 118-121.
10. S. Chapman, "The Lunar Atmospheric Tide at Greenwich, 1854-1917," *Quarterly Journal of the Royal Meteorological Society*, vol. 44, 1918, pp. 271-279.
11. H. H. Clayton, *World Weather*, The Macmillan Co., New York, 1923, 393 pp.
12. A. Defant, "Die Veränderungen in der allgemeinen Zirkulation der Atmosphäre in den gemässigten Breiten der Erde," *Sitzungsberichte, Kaiserliche Akademie der Wissenschaften*, Wien, vol. 121, Mar. 1912, pp. 379-586.
13. L. Descroix, "Sur l'amplitude et la durée moyenne des oscillations extrêmes du baromètre à Paris," *Comptes Rendus*, vol. 116, No. 23, 1893, pp. 1320-1321.
14. M. F. Freeman and J. W. Tukey, "Transformations Related to the Angular and the Square Root," *Annals of Mathematical Statistics*, vol. 21, 1950, pp. 607-611.
15. R. W. Hamming and J. W. Tukey, *Measuring Noise Color*, (Unpublished).
16. B. Haurwitz, "The Motion of the Atmospheric Disturbances on the Spherical Earth," *Journal of Marine Research*, vol. 3, No. 3, 1940, pp. 254-267.
17. L. F. Hubert, "Analysis Aids for the American Tropics," *Monthly Weather Review*, vol. 86, No. 6, June 1958, pp. 201-218.
18. H. Landsberg, *Physical Climatology*, 2d Ed., Gray Printing Co., Inc., Dubois, Pa., 437 pp., (426-437).
19. I. Langmuir, "A Seven-Day Periodicity in Weather in the United States During April, 1950," *Bulletin of the American Meteorological Society*, vol. 31, No. 10, Dec. 1950, pp. 386-387.
20. W. Meinardus, "Die Luftdruckverhältnisse und ihre Wandlung südlich von 30°S Br.," *Deutsche Südpolarexpedition 1901-1903*, vol. 3, pt. 3, 307 pp. [1928].
21. R. Merecki, "Die Sonnentätigkeit und die unperiodischen Luftdruckänderungen," *Meteorologische Zeitschrift*, vol. 21, No. 1, Jan. 1904, pp. 1-18.

22. R. Merecki, "Über den Einfluss der veränderlichen Sonnen-tätigkeit auf den Verlauf der meteorologischen Elemente auf der Erde," *Meteorologische Zeitschrift*, vol. 27, No. 2, Feb. 1910, pp. 49-61.
23. H. A. Panofsky and G. W. Brier, *Some Applications of Statistics to Meteorology*, The Pennsylvania State University, University Park, Pa., 1958, 224 pp.
24. H. A. Panofsky and R. A. McCormick, "Properties of Spectra of Atmospheric Turbulence at 100 Meters," *Quarterly Journal of the Royal Meteorological Society*, vol. 80, No. 346, 1954, pp. 546-564.
25. W. Portig, "Erste Mitteilung über schnell-laufende globale Luftdruckwellen," *Meteorologische Rundschau*, vol. 10, 1957, pp. 54-58.
26. C.-G. Rossby and collaborators, "Relations between Variations in the Intensity of the Zonal Circulation of the Atmosphere and the Displacements of the Semi-permanent Centers of Action," *Journal of Marine Research*, vol. 2, No. 1, 1939, pp. 38-55.
27. B. Saltzman, "Some Hemispheric Spectral Statistics," *Journal of Meteorology*, vol. 15, No. 3, June 1958, pp. 259-263.
28. J. Scoles, cited after J. Hann, "Zum Klima von Malta" *Meteorologische Zeitschrift*, vol. 20, No. 2, 1903, pp. 73-74. (Original data in: *Results of Meteorological and Magnetical Observations*, Appendix, Stoneyhurst College Observatory, 1892.)
29. R. S. Scorer, "The Use of Normals in Numerical Forecasting," *Tellus*, vol. 6, No. 1, Feb. 1954, pp. 23-31.
30. G. C. Simpson, *Meteorology, British Antarctic Expedition 1910-1913*, vol. I, Calcutta, 1919.
31. J. Smagorinsky, "General Circulation Experiments with the Primitive Equations," Paper presented before AAAS-AMS Meeting, December 1958. For abstract see: *Bulletin of the American Meteorological Society*, vol. 39, No. 11, Nov. 1958, p. 607.
32. H. J. Stewart, "Kinematics and Dynamics of Fluid Flow," Sect. VI, in *Handbook of Meteorology*, Berry, Bollay, and Beers, Editors, McGraw-Hill Book Co. Inc., New York, 1945, pp. 412-498.
33. K. Takahashi and collaborators, "Long Periodic Oscillations of the Atmosphere Between Low and High Latitudes," [Abstract], *Scientific Proceedings*, Rome Meeting 1954, International Union of Geodesy and Geophysics, Association of Meteorology, London, 1956, p. 358.
34. H. C. S. Thom, Personal communication.
35. J. W. Tukey, "The Sampling Theory of Power Spectrum Estimates," *Symposium on Applications of Autocorrelation Analysis to Physical Problems*, Woods Hole, Mass., 13-14 June 1949, Office of Naval Research, Washington, D.C., 1950, pp. 47-67.
36. S. W. Visser, "The 27-Day Period in United States Temperature," *Transactions of the American Geophysical Union*, vol. 39, No. 5, Oct. 1958, pp. 835-844.
37. A. Wegener, "Meteorologische Terminbeobachtungen am Danmarks-Havn," *Danmark Ekspeditionen til Grønlands Nord-østkyst, 1906-1908*, vol. 2, No. 4, Copenhagen, 1911, p. 316.
38. L. W. Wing, "Ultra Long Waves and Solar Terrestrial Cycle Relationships," *Journal of Cycle Research*, vol. 6, Nos. 3-4, 1957, pp. 55-119.